

Exotic phases in compact stars

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Abstract. We discuss how the co-existence of hyperons, antikaon condensate and color superconducting quark matter in neutron star interior influences the gross properties of compact stars such as, the equation of state and mass-radius relationship. We compare our results with the recent observations. We also discuss about superdense stars in the third family branch which may contain a pure color-flavor-locked (CFL) core.

1. Introduction

Recent developments in dense matter physics indicates that at sufficiently high density there is a possibility of phase transition from hadronic matter to color-flavor-locked quark matter. This quark matter is different from the unpaired quark matter in the way that quarks with all three flavors and colors form Cooper pairs near their Fermi surfaces as their interaction is attractive in the color antisymmetric channel. The formation of diquark condensates breaks the color gauge symmetry [1, 2, 3, 4, 5, 6, 7]. The CFL configuration of the 3 flavor quarks is believed to be the true ground state of matter at very high density.

The CFL phase is of special interest in the context of compact stars, where the density may rise up to a few times normal nuclear matter density. In the high-density core, recently there have been studies on nuclear-CFL quark matter phase transition [8, 9, 10] and its impact on the structure of dense star. Also, the structure of compact star including pure CFL quark matter have been studied by others [11]. The color superconducting phase in the interior of compact stars has also been constructed in the Nambu-Jona-Lasino model [12, 13]. In this paper, we study a phase transition of hadronic matter to CFL quark matter and its impact on the structure of compact stars. Along with CFL matter, hyperons and K^- condensate may also co-exist in the compact star interior. We investigate the effect of all three forms of exotic matter on the equation of state (EoS) and mass-radius profile of dense stars. The theoretical investigation of mass-radius profile of stars is important because this can be directly compared to observations, which leads to determination of the correct EoS and compositions of compact stars core.

2. Formalism

To describe the hadronic phase, we employ the density-dependent relativistic hadron (DDRH) model [14, 15, 16] for baryon-baryon interaction which is given by the

Lagrangian density

$$\begin{aligned}\mathcal{L}_B = \sum_B \bar{\Psi}_B & \left(i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho B} \gamma_\mu \boldsymbol{\tau}_B \cdot \boldsymbol{\rho}^\mu + \frac{1}{2} g_{\delta B} \boldsymbol{\tau}_B \cdot \boldsymbol{\delta} \right) \Psi_B \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) + \frac{1}{2} (\partial_\mu \delta \partial^\mu \delta - m_\delta^2 \delta^2) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} \\ & + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu.\end{aligned}$$

The interactions among baryons are mediated by the exchange of σ , ω , ρ and δ mesons. Here meson-baryon vertices are dependent on vector density and determined using Dirac-Brueckner-Hartree-Fock (DBHF) calculations. In addition to all the species of the baryon octet, we consider antikaon condensation of K^- mesons in the hadronic phase and the Lagrangian density for (anti)kaon condensation in the minimal coupling scheme is, $\mathcal{L}_K = D_\mu^* \bar{K} D^\mu K - m_K^{*2} \bar{K} K$, $D_\mu = \partial_\mu + i g_{\omega K} \omega_\mu + i g_{\rho K} \boldsymbol{\tau}_K \cdot \boldsymbol{\rho}_\mu / 2$ being the covariant derivative and m_K^* , the effective mass of antikaons given by $m_K^* = m_K - g_{\sigma K} \sigma - \frac{1}{2} g_{\delta K} \boldsymbol{\tau}_{3\bar{K}} \cdot \boldsymbol{\delta}$ [17, 18].

We perform our calculations in the mean field approximation and the density-dependent meson-nucleon vertices are obtained from microscopic Dirac-Brueckner calculations of symmetric and asymmetric nuclear matter using Groningen nucleon-nucleon potential [15]. The meson-hyperon vertices are also made density dependent using hypernuclear data [19] and the scaling law [20]. However, meson-(anti)kaon couplings remain density-independent.

The pure quark matter is composed of paired quarks of all flavors and colors and neutral kaons which are Goldstone bosons arising due to the breaking of chiral symmetry in the CFL phase [21]. Color neutrality is imposed in the CFL phase and this automatically leads to electric charge neutrality [22, 23]. The free energy of the CFL quark matter to order Δ^2 is given by [8, 10, 21] $\Omega_{CFL}^q = \frac{1}{\pi^2} \sum_{i=1}^9 \int_0^\nu p^2 (\sqrt{p^2 + m_i^2} - \mu_i) dp - \frac{3\Delta^2 \mu^2}{\pi^2} + B$, where Δ is the gap and B is the bag constant, μ and ν are average chemical potential and common Fermi momentum of quarks related as $\nu = 2\mu - \sqrt{\mu^2 + \frac{m_s^2}{3}}$, μ_i and m_i are chemical potential and mass of quarks respectively.

The quark number densities are $n_u = n_d = n_s = \frac{(\nu^3 + 2\Delta^2 \mu)}{\pi^2}$. And, the electron chemical potential (μ_e) vanishes as the CFL matter is electric charge neutral. There is also a possibility of K^0 condensation [24, 25] in the CFL matter through which it relaxes under stresses such as non-zero strange quark mass and electron chemical potential. The thermodynamic potential ($\Omega_{CFL}^{K^0}$) due to K^0 condensate is given by Ref.[24]. The pressure in the CFL+ K^0 phase is given by $P^{CFL} = -\Omega_{CFL}^q - \Omega_{CFL}^{K^0}$ and the energy density (ϵ^{CFL}) is obtained from the Gibbs-Duhem relation.

We have considered a first order phase transition from hadronic matter to color-flavor-locked matter. The mixed phase of hadronic matter consisting of hyperons and antikaon condensate and the CFL phase is governed by Gibbs phase rules which read, $P^h = P^{CFL}$ and $\mu_n = 3\mu$ where μ_n is chemical potential of neutron. The condition of baryon number conservation $n_b = (1 - \chi)n_b^h + \chi n_b^{CFL}$ is maintained globally, where χ is the volume fraction of CFL phase in the mixed phase and n_b^h and n_b^{CFL} are baryon densities in the hadronic and CFL phase respectively. However, global charge neutrality condition is relaxed and electric charge neutrality in the hadronic phase and color charge neutrality in the CFL phase are imposed locally [9]. The total energy density in the mixed phase has contribution from both the hadronic and CFL phases and is given by $\epsilon = (1 - \chi)\epsilon^h + \chi\epsilon^{CFL}$.

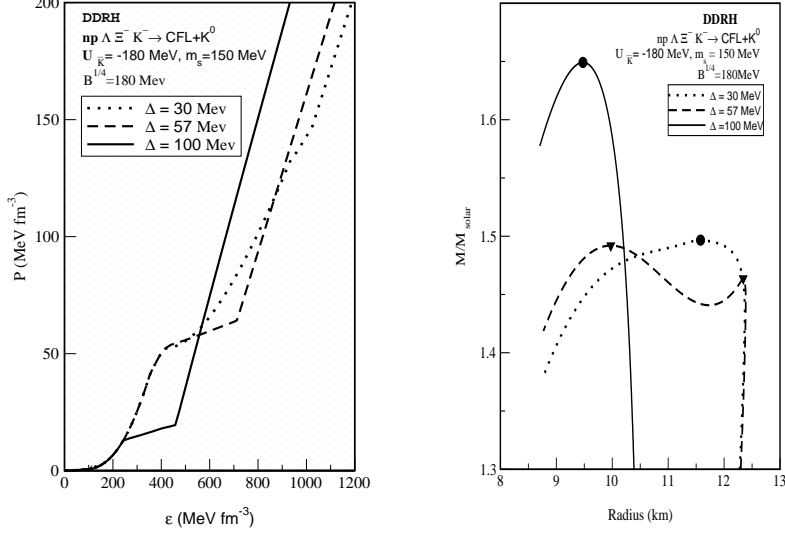


Figure 1. a. The equation of state, pressure P versus energy density ϵ , for for n, p, Λ , Ξ , lepton, K^- and CFL+ K^0 s is shown here for $m_s = 150$ MeV, $B_{\frac{1}{4}} = 180$ MeV. b. The mass-radius relationship for the corresponding EoS of Figure 1a.

3. Results & Discussions

The equation of state (EoS) for compact star matter with hadronic phase consisting of K^- condensate in addition to hyperons are exhibited in Figure 1a for the parameter set of $B_{\frac{1}{4}} = 180$ MeV, $m_s = 150$ MeV, and $U_{\bar{K}}(n_0) = -180$ MeV but for a range of possible values of Δ (30-100 MeV) [See: Table-1]. The two boundaries of mixed phase is identified by the two kinks. For lower Δ values the mixed phase appears at higher density, for example when $\Delta = 30$ MeV it starts at $4.73n_0$ and continues for a short density range up to $5.19n_0$, while for $\Delta = 57$ MeV CFL sets in earlier at $2.27n_0$ and terminates at $3.94n_0$. Here, Λ and Ξ^- appear before mixed phase, while K^- condenses before the mixed phase for $\Delta = 30$ MeV and in the mixed phase for $\Delta = 57$ MeV. But it is interesting to note that the early appearance of CFL phase for $\Delta = 100$ MeV at $1.43n_0$ does not allow other exotic components of matter such as hyperons and K^- condensate to appear in the system. Here the hadronic phase essentially consists of nucleons only. The early commencement of CFL phase for $\Delta = 100$ MeV though makes the EoS soft at the lower density regime, the overall EoS in this case is quite steep. In fact this EoS is the steepest of all the cases discussed here and has considerable effect on the structure of the star. However, K^0 condensate in the CFL phase does not have any significant contribution towards the energy density term.

In Figure 1b mass-radius profile has been displayed for the corresponding EoS of Figure 1a. The filled patterns correspond to the maximum masses, the values are given in Table I. It is observed that softer the overall EoS, lower is the maximum mass. Our results of mass-radius are consistent with the recent observations by Chandra X-

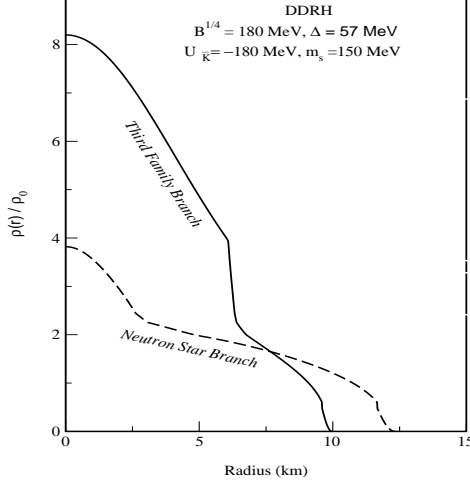


Figure 2. Density profile of the maximum mass stars in the neutron star and third family branch with n, p, Λ , Ξ , lepton, K^- ($U_{\bar{K}}(n_0) = -180$ MeV) and CFL+ K^0 matter with $B^{\frac{1}{4}} = 180$ MeV, $m_s = 150$ MeV, $\Delta = 57$ MeV.

ray observatory and Hubble Space Telescope (HST) on the isolated neutron star RX J185635-3754. Analysis of Chandra data gives an radiation radius of $\sim 10 - 12$ km [26], while HST data predict a radius $R = 11.4 \pm 2$ km [27]; these observations can be well explained with compact stars having a hadronic matter consisting of all three forms of exotic matter i.e. hyperons, Bose-Einstein condensate of K^- mesons and CFL quark matter. Also, the values of maximum masses, which we have obtained, are in good agreement with the Hulse Taylor pulsar of mass $1.44 M_{\text{solar}}$.

Moreover, we have found a stable sequence of superdense stars beyond the neutron star branch for the parameter set of $B^{\frac{1}{4}} = 180$ MeV, $m_s = 150$ MeV and $U_{\bar{K}}(n_0) = -180$ MeV and $\Delta = 57$ MeV. The second set of solutions are called the third family of compact stars [28]. It is interesting to note that the radii in the third family branch are smaller than their counterparts in neutron star branch.

The density-radius profiles of the maximum mass stars corresponding to the EoS that yields two stable maxima are shown in Figure 2, the maximum masses being $1.464 M_{\text{solar}}$ and $1.492 M_{\text{solar}}$ for the neutron star and third family branch respectively. We find that the maximum mass neutron star of radius 12.38 km has a mixed hadronic-CFL matter core up to a radius of 3 km. The third family maximum mass star, on the other hand, has a pure CFL matter core up to 6.39 km out of total 9.96 km radius. It is also interesting to note that the variation of density with radius is very fast in the third family branch unlike that of the neutron star. The rise of density in the third family branch is steep but continuous.

In figure 3 we compare the distribution of particles in two stars-one from the

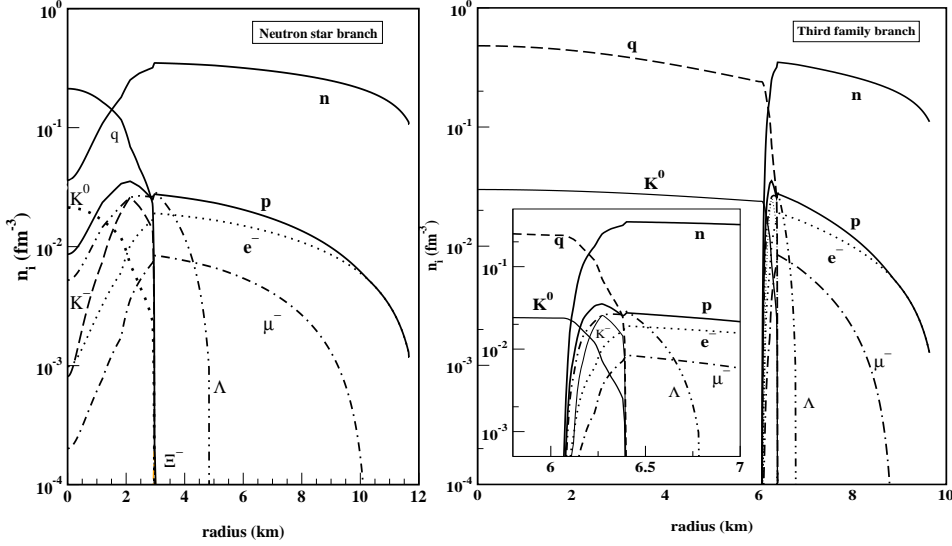


Figure 3. Compositions of (a.) a neutron star (b.) a third family star

neutron star branch and another from the third family branch and observe that they have got quite different compositions. The neutron star interior contains a mixed phase which is mainly dominated by CFL+ K^0 , though contains a considerable fraction of n, p, hyperons, antikaon condensate and leptons (Figure 3a). The core of the third family star however is made up of pure CFL+ K^0 matter. The hadronic matter here disappears very sharply as soon as CFL phase starts; the distribution of particles in the mixed phase can be viewed from the inset picture of Figure 3b, it exists for a narrow radius strip. It is noted that Ξ^- appears just before the onset of the mixed phase and lepton density drops. But Ξ^- fraction itself drops very soon as CFL phase sets in. K^- condense in the mixed phase which rapidly replaces leptons and grows fast, causes proton fraction to rise. But with the rising CFL fraction, all the hadrons melt into quarks and the core consists of CFL+ K^0 matter only. However, the crusts of both the branches contain n, p and leptons.

4. Summary

We have studied a first order phase transition from hadronic matter to CFL quark matter. The hadronic part has been described in the framework of DDRH model and some exotic forms of matter such as hyperons and antikaon condensate have been considered in this phase. The CFL phase contains Goldstone boson K^0 . We have found that the early appearance of any of the exotic matters delays the onset of others. Also, a smaller value of gap supports softer EoS leading to smaller maximum mass stars. We have obtained a stable sequence of compact stars beyond the known neutron stars branch, that we call the third family stars. The third family stars contain a pure CFL+ K^0 core, while the neutron stars have a mixed hadronic and CFL+ K^0 core. We have also shown that the compact stars in the third family have smaller radii

Table 1. Lower(l) and upper(u) boundaries of the mixed phase for energy-density (ϵ) and density ($u = n/n_0$) in hadron-CFL phase transition for different values of gap $\Delta = 30, 57, 100$ MeV for a given value of bag constant $B^{\frac{1}{4}} = 180$ MeV, $m_s = 150$ MeV and saturation density $n_0 = 0.18 fm^{-3}$. The maximum neutron star and third family star masses M_{max}/M_{solar} , their radii and corresponding central densities $u_{cent}=n_{cent}/n_0$ are shown below.

Δ MeV	Mixed Phase				Neutron star			Third family		
	ϵ_l MeV fm^{-3}	ϵ_u MeV fm^{-3}	u_l	u_u	u_{cent}	$\frac{M_{max}}{M_{solar}}$	radius km	u_{cent}	$\frac{M_{max}}{M_{solar}}$	radius km
30	930.52	1022.25	4.73	5.19	4.87	1.500	11.58			
57	406.87	711.50	2.27	3.94	3.82	1.464	12.34	8.20	1.492	9.97
100	245.63	457.67	1.43	3.09	9.13	1.649	9.48			

compared to their counterparts in the neutron star branch.

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